Transition prediction in incompressible boundary layer with finite-amplitude streaks

6th Symposium on Global Flow Instability and Control
29 September 2015, Hersonissos, Crete, Greece

Juan A. Martin and Pedro Paredes

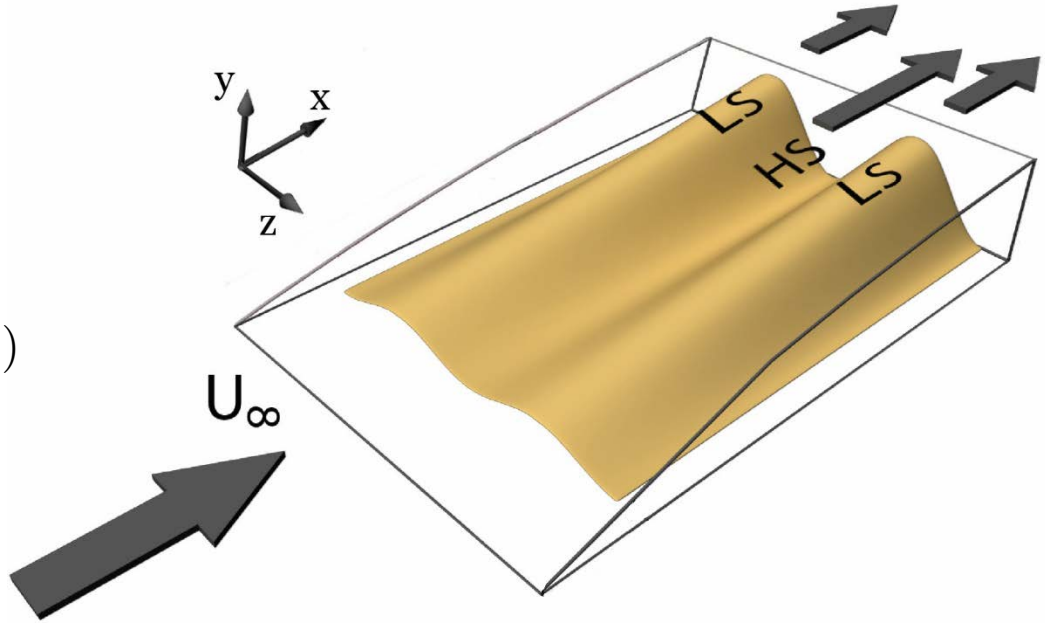
juanangel.martin@upm.es
Depto. de Matemática Aplicada a la Ingeniería Aeronáutica
ETSI Aeronáutica y del Espacio, Universidad Politécnica de Madrid
28040 Madrid, Spain

Introduction

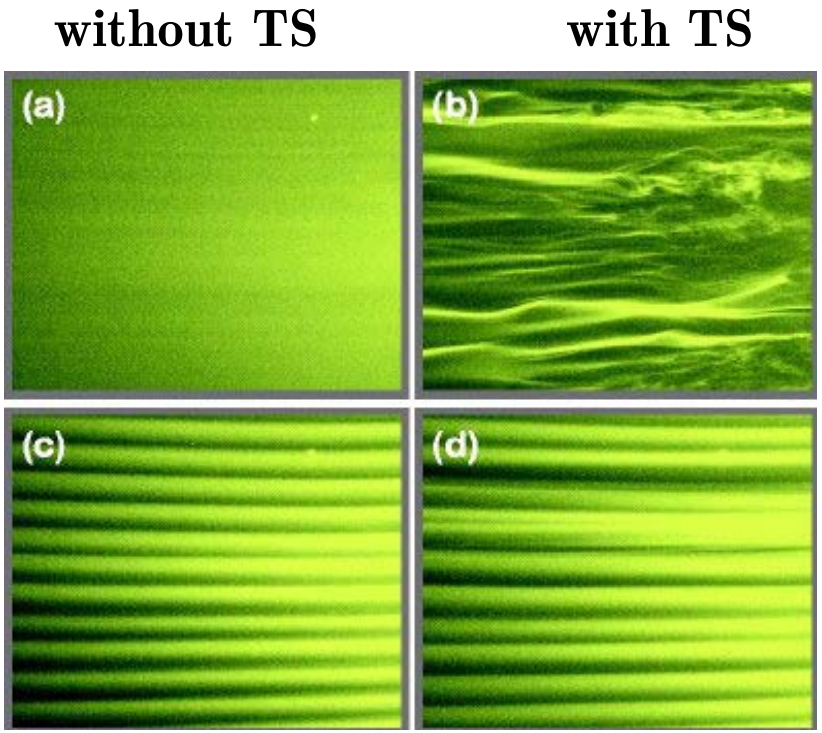
- Research efforts focused on transition delay

- Strategy: modulate boundary layer velocity profile
- Introducing streamwise streaks

  - 3D flow structures
  - streamwise long,
    wall-normal and spanwise thin
  - alternating regions of high (HS)
    and low (LS) speed



Estabilizing effect of streaks



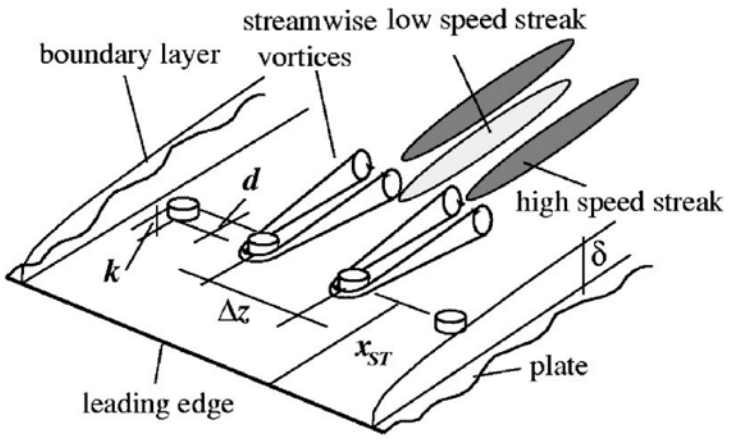
Fransson, Talamelli, Brandt & Cossu, PRL 2006

Estabilizing effect increased with streak amplitude until secondary instability developes for high intensity streaks

Andersson & al., JFM 2001

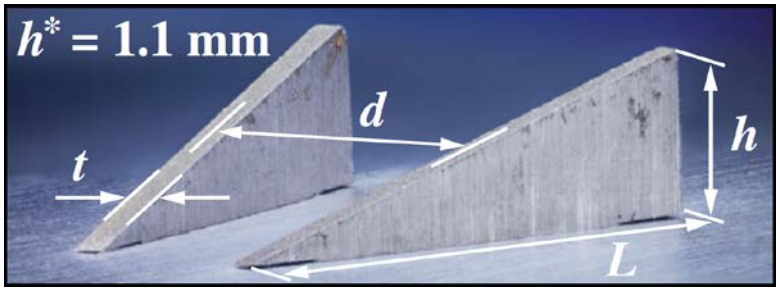
Streak generation

Roughness elements



Fransson, Brandt, Talamelli & Cossu, PoF 2005

Miniature Vortex Generators (MVG's)



Shahinfar, Sattarzdeh, Fransson & Talamelli, PoF 2012

$$A_s = \frac{1}{2U_\infty} \max_y \left[\max_z \left(U(y,z) \right) - \min_z \left(U(y,z) \right) \right]$$

~12%

~32%

Motivation to compute high intensity streaks

Formulation

- Objective: cheap CPU method

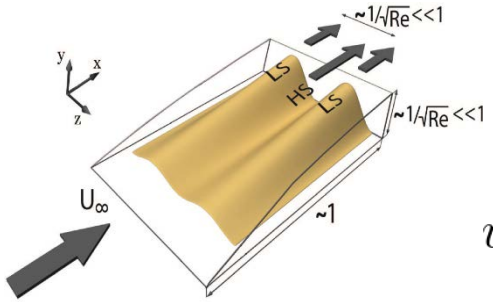
  Incompressible, steady, streaky base flow

➡ Reduced Navier Stokes equations (RNS)

» Simplified boundary layer-like formulation (**BRE's**) – *Fletcher 1990*

» High Reynolds number limit $Re = U_\infty L/\nu \gg 1$

» Two short scales, one long



$$
\begin{aligned}
u_x + v_y + w_z &= 0 \\
uu_x + vu_y + wu_z &= dP_\infty/dx + u_{yy} + u_{zz} \\
uv_x + vv_y + wv_z &= p_y + v_{yy} + v_{zz} \\
uw_x + vw_y + ww_z &= p_z + w_{yy} + w_{zz}
\end{aligned}
$$

» **Parabolic in x**

» **Marching scheme, 2nd order accuracy**

» **Fast and robust**

» **Re-independent**

Formulation

- Objective: cheap CPU method

  Incompressible, steady, streaky base flow

➡ Reduced Navier Stokes equations (RNS)

Stability analysis

➡ 3D Parabolized Stability Equations (PSE-3D)

» 3D linear stability equations for base flows with a single slowly-varing spatial direction

$$\mathbf{q} = \bar{\mathbf{q}} + \varepsilon \tilde{\mathbf{q}} \qquad \tilde{\mathbf{q}}(x,y,z,t) = \sum_{n=-N}^{N} \breve{\mathbf{q}}_n(x,y,z) \exp\left[-in\omega t\right] \qquad \breve{\mathbf{q}}(x,y,z) = \hat{\mathbf{q}}(x,y,z) \exp\left[\mathrm{i} \int_x \alpha(x')dx'\right]$$

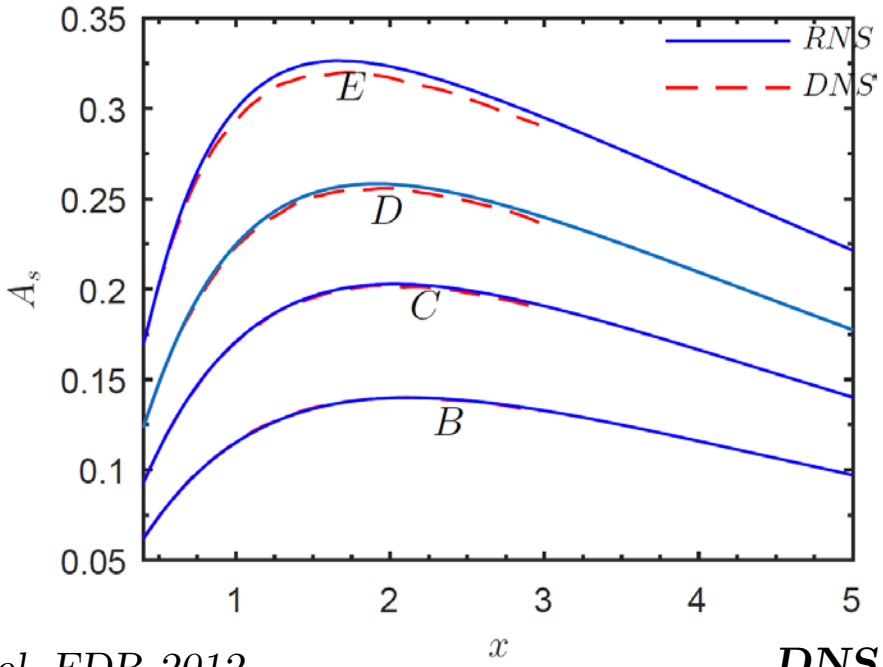
» Initial condition: Spatial Bi-Global analysis

RNS results: nonlinear streaks

- Downstream evolution of optimal streaks

$\beta = 0.45$ *Andersson & al., PoF 1999*

$$A_s(x) = \frac{1}{2}(\max_{y,z}(u - U_b) - \min_{y,z}(u - U_b)),$$



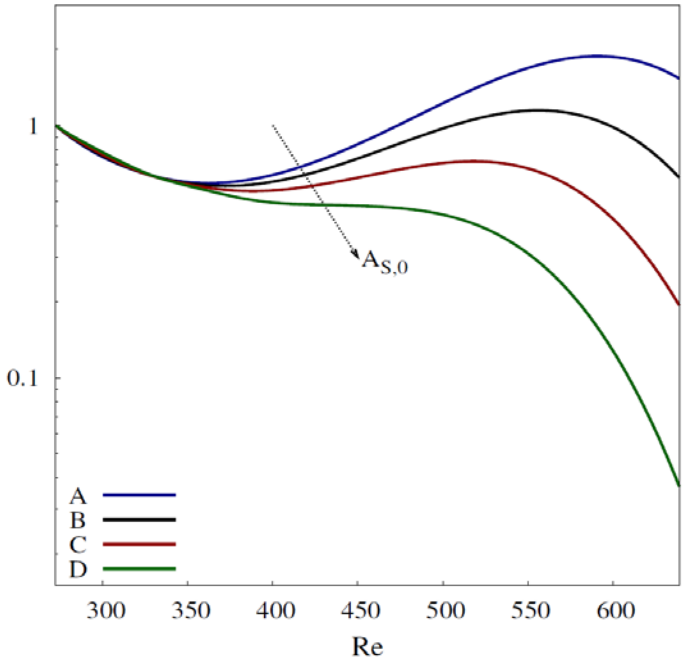
RNS *Martin & Martel, FDR 2012*

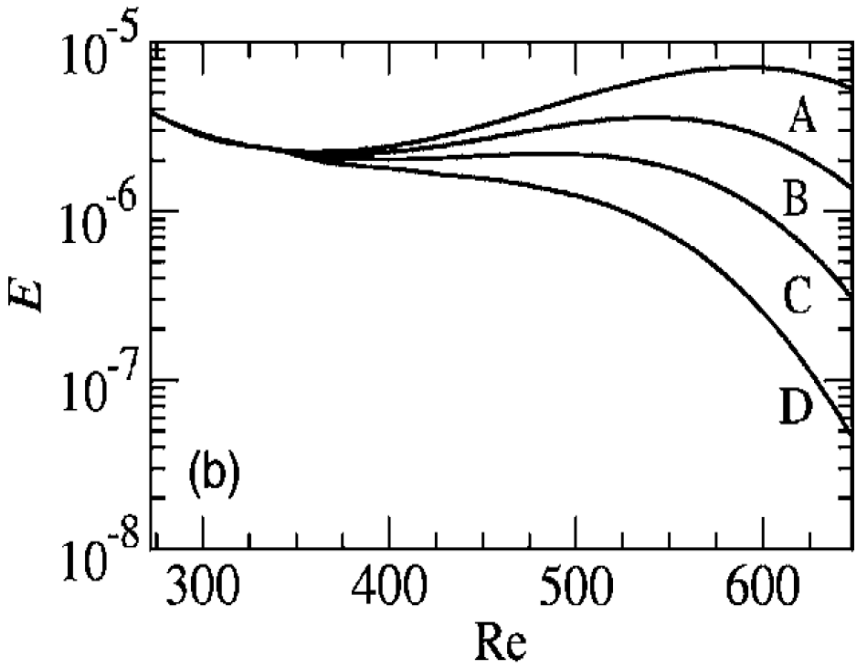
DNS *Cossu & Brandt, PoF 2002*
EJFM/B 2004

PSE-3D results: TS energy

- Kinetic energy evolution of a TS wave

» $w = 0.0358$

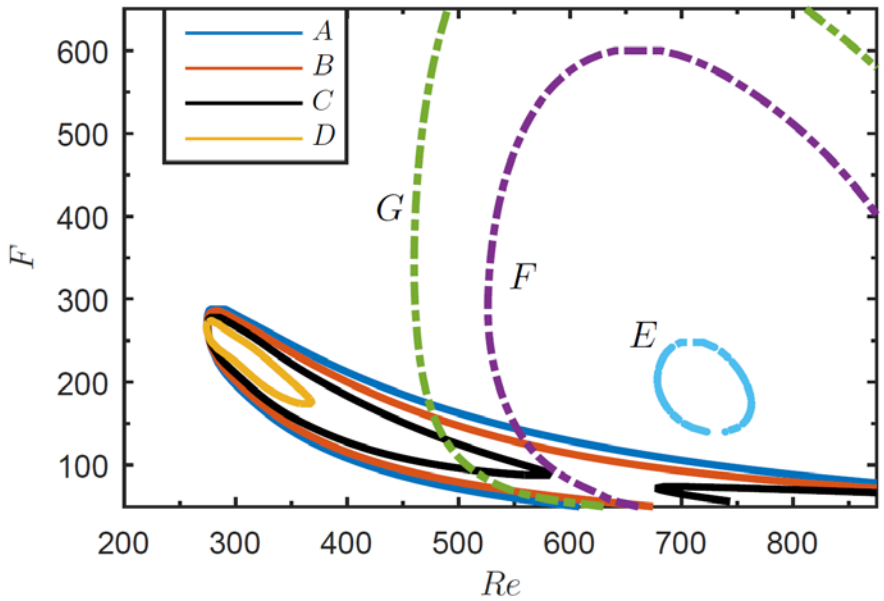




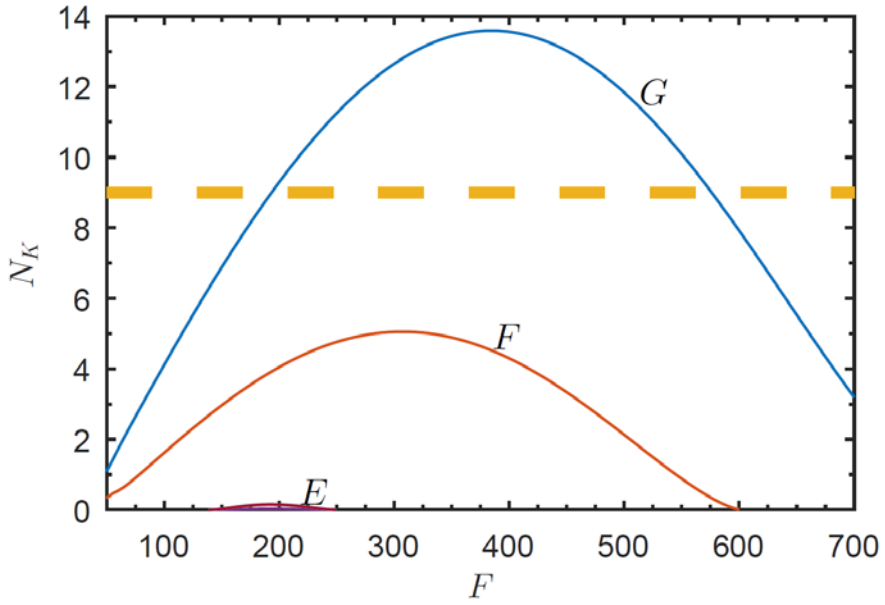
PSE-3D results

Bagheri & Hanifi, PoF 2007

PSE-3D streak stability analysis



Neutral stability curves



***N*-factor plot for secondary unstable streaks**

$Re = 400$

TS mode

$\mathbf{A_{s\,max}(A) = 0\%}$
$\mathbf{A_{s\,max}(B) = 14\%}$
$\mathbf{A_{s\,max}(C) = 20\%}$
$\mathbf{A_{s\,max}(D) = 25\%}$

Critical streak amplitude
$\mathbf{A_{s\,max}}$ **~26%**

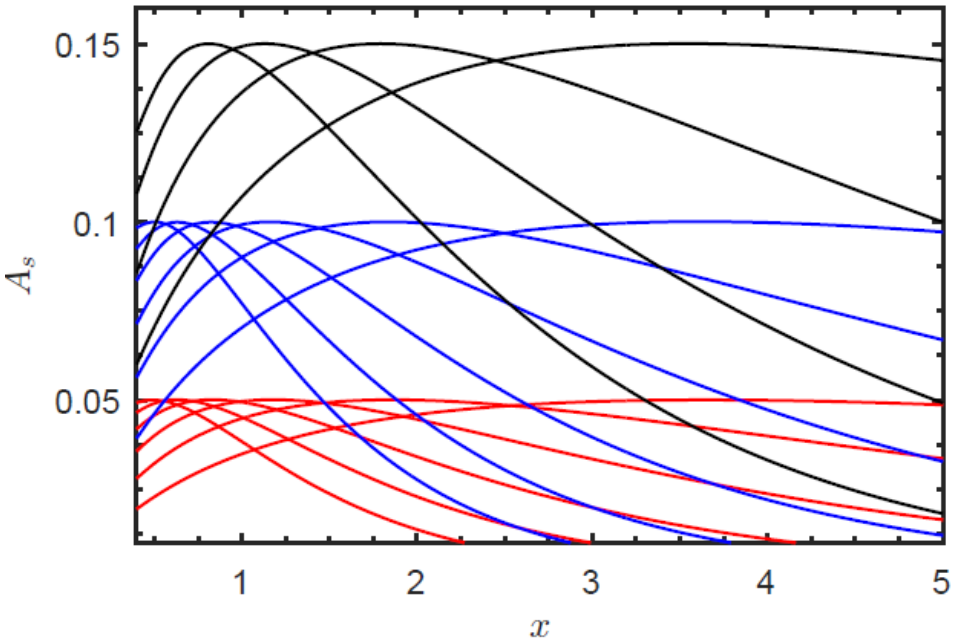
Andersson & al., JFM 2001

$\mathbf{A_{s\,max}(E) = 28\%}$
$\mathbf{A_{s\,max}(F) = 32\%}$
$\mathbf{A_{s\,max}(G) = 36\%}$

shear layer mode

Parametric analysis optimal streaks

- Varying spanwise wave number β
- Optimization criteria: max energy at $x = 1$
- Gives the velocity profile for initial condition
- Family of streaks with same maximum intensity



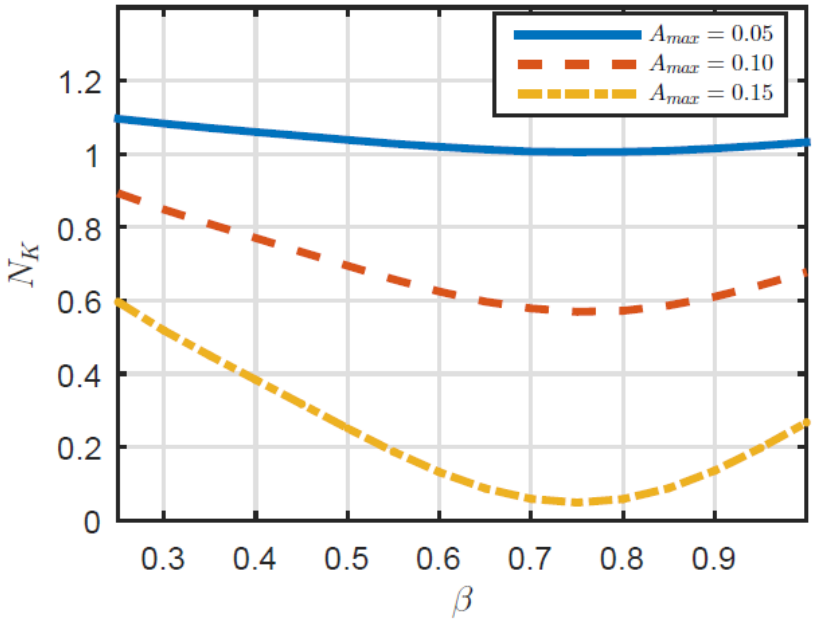
$\mathbf{A_{s\ max} = 15\%}$

$\mathbf{A_{s\ max} = 10\%}$

$\mathbf{A_{s\ max} = 5\%}$

Parametric analysis optimal streaks

- Varying spanwise wave number β
- Optimization criteria: max energy at $x = 1$
- Gives the velocity profile for initial condition
- Family of streaks with same maximum intensity



» **Existence of β that minimizes energy growth** *(results inline with NPSE analysis of Bagheri & Hanifi PoF 2007)*

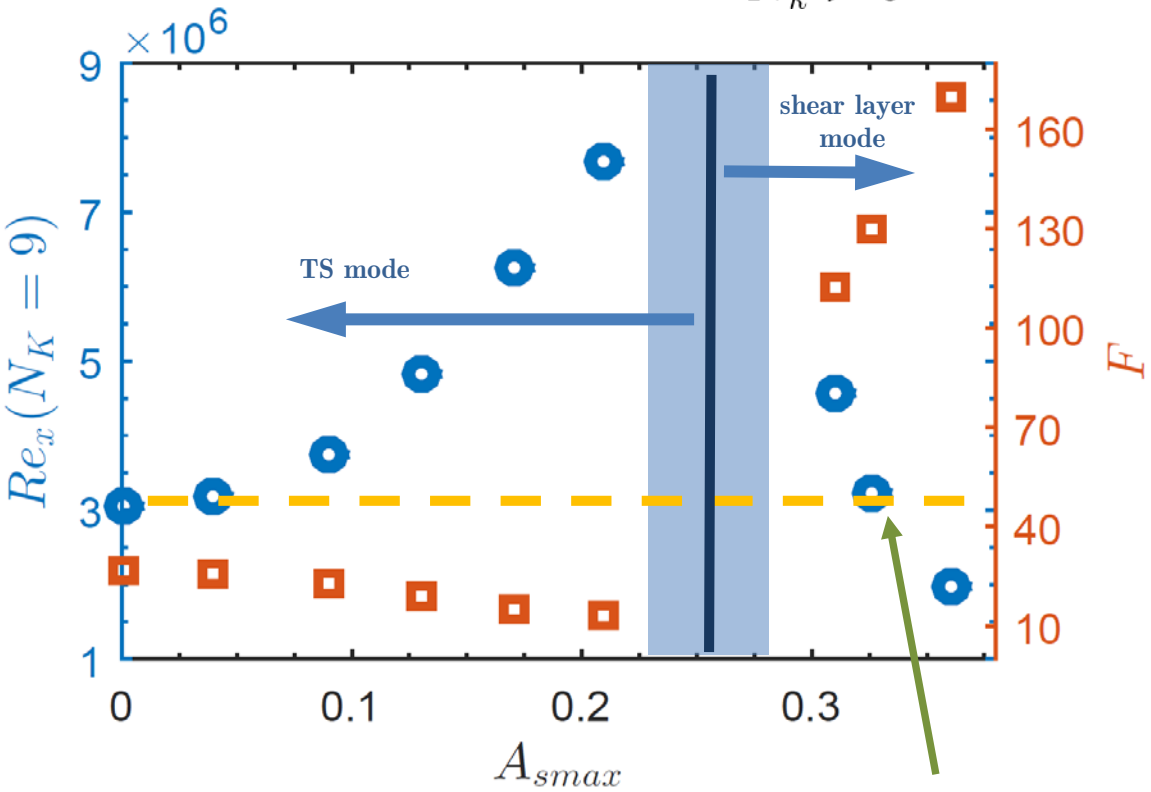
» **Kept constant with streak amplitude**

Effect on transition location

- Based on N-factor as $N_k = \ln\left(K(x_{ii})/K(x_i)\right)$
- Transition is assumed to occur when $N_k > 9$

$Re = 1000$

$Re_x = Re^2$



Estimation of maximum streak amplitude for delaying transition

$\mathbf{A_{s\ max}}$ **~33%**

Concluding remarks

- RNS to describe the downstream evolution of streaks
- PSE-3D to analyze the stability of streaks
- Cheap CPU method – desktop Workstation
  - » ~ minutes to obtain streaky flow
  - » ~ hours to compute a neutral curve (40 frequencies)
  - » ~ days to do the complete parametric study
- Neutral stability curves of 3D-modified TS and shear layer streak instabilities
- Optimum spanwise wavenumber for TS damping (constant with streak intensity)
- Displacement of transition location with streak intensity

» *Reference: Martin & Paredes, Transition prediction in incompressible boundary layer with finite-amplitude streaks, Journal of Fluid Mechanics (in preparation).*

- Thanks for your attention!!

  – Questions??